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A Cross-Age Study of Students' Understanding of Limit and Continuity Concepts

A Compreensão dos Conceitos de Limite e Continuidade: um estudo desenvolvido com alunos em distintos momentos de um curso de formação inicial para professores

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Abstract

The aim of this study is to reveal concept development and the way limit and continuity concepts are understood by students from different levels of education. For this purpose, a test comprising open-ended questions about verbal, algebraic and graphical representations of concepts was administered to students from different levels of education. When students' understandings of limit and continuity concepts are compared, the pre-service teachers in their 3rd year of study were found much less successful than other students in algebraic, verbal and graphical representations of limit and continuity concepts. It may be recommended that when designing instructional activities verbal, graphical and algebraic representations should be prioritized to enhance the development of students' interpretation skills of different representations of functions.

Keywords: Mathematics Education. Limit and Continuity Concepts. Cross-Age Study

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Resumo

O objetivo deste artigo é analisar como os conceitos de limite e continuidade são compreendidos por estudantes em diferentes momentos de formação. Para isso, foi aplicado a esses alunos um teste composto por questões abertas no qual foram privilegiadas as representações verbais, algébricas e geométricas (gráficas) de funções. O estudo das compreensões manifestadas nos testes revela que os estudantes, futuros professores, em seu terceiro ano de formação, apresentam maiores problemas que os demais alunos quanto aos conceitos em questão. Disso decorre a recomendação de que, quando elaborando atividades instrucionais, representações verbais, visuais e geométricas (gráficas) devem ser priorizadas de modo a viabilizar o desenvolvimento de estratégias interpretativas adequadas que permitam trabalhar com diferentes representações de funções.

Palavras-chave: Educação Matemática. Limite e Continuidade. Estudo comparativo transversal (por idade).

Introduction

Since 'Limit and Continuity' concepts are crucial for learning advanced mathematical concepts, much attention has been paid to them in mathematics education research (MASTORIDES; ZACHARIADES, 2004). These two concepts are also pre-requisites for learning many mathematical concepts. For example, unless a function is continuous at a given point, it is not differentiable there, thereby, differentiability counts on continuity of functions. To show whether a given function is not differentiable is enough to indicate that the function is not continuous. Hence, the concepts 'limit and continuity' are central for further mathematics learning, i.e., analysis, differential equations, differential geometry, and advanced mathematical statistics. Also, the concept 'limit' is a prerequisite for an understanding of the real numbers in 'Calculus' (BURN, 2005; BERGE, 2006).

Students' understanding of limit and continuity concepts

Previous research indicates that limit and continuity concepts are among the most challenging concepts for students and pre-service teachers (CORNU,

1991; FERRINI-MUNDY; GRAHAM, 1991; MAMONA-DOWNS, 2001; SZYDLIK, 2000; TALL; VINNER, 1981; DAVIS; VINNER, 1986; SIERPINSKA, 1987). These studies have concluded that majority of their students had difficulty in understanding limit, continuity and the related concepts. Particularly the concept of limit is not an easy to understand concept due to containing operations such as infinite (ÖZMANTAR; YEŞILTEPE, 2008). Why students fail to address meaningful ideas on role of the concept 'limit' may stem from their inappropriate and weak mental links between knowledge of limit and that of other calculus concepts such as continuity, derivative and integral (BEZUIDENHOUT, 2001). Students' conceptions of functions link with their conceptions of limits and continuity. Bezuidenhout (2001) reports that students attempt to determine the continuity of a function at a point by looking at whether the function is defined or not at that point. Moreover students express that if the limit exists, the function will be continuous at that point. A similar result was found by Vinner (1992) in his work with 406 college calculus students. Whether the functions are presented graphically or symbolically, many students expressed their belief that for a function to be continuous is the same as being defined and to be discontinuous is the same as being undefined at a certain point.

In addition some teachers and students strongly tend to view continuous functions as one expressed formula (TALL; VINNER, 1981; HITT, 1994; HITT, 1998; HITT; PLANCHART, 1998). In the work of Akbulut and Işık (2005) with 100 pre-service elementary mathematics teachers, 12 of the participants described limit as a point or place where a function can never reach. The result of Williams' study (1991) conducted with university students is similar. In their study with secondary mathematics teachers, Mastorides and Zacharlades (2004) determined that the majority of teachers did not have deep understandings about limit and continuity concepts. Furthermore, it was found that some teachers could not read graphics of functions and transfer a verbal expression to symbolic expression correctly.

The role of representations for understanding limit and continuity concepts

The role of representations in mathematics education has emerged in recent literature as fundamental aspect in facilitating students' construction of mathematical concepts as well as their problem solving abilities (JANVIER, 1987; SCHOENFELD, 1994; ZIMMERMANN; CUNNINHAM, 1991). For a deeper conceptual understanding of Calculus, instructions should be focused not only on the use of algebraic representations but additionally should take into account the geometric and intuitive representations of the corresponding mathematical objects as well as the interaction among these multiple representations (KAPUT; 1994). Kaput (1992) claims that the use of more than one representation or notation system helps students to obtain a better picture of a mathematical concept. Thus, the ability to identify and represent the same concept through different representations is considered as a prerequisite for the understanding of the particular concept (DUVAL, 2002; EVEN, 1998). Students' level of knowledge about a function and their interpretation skills of different representations of functions have a key role in the development of limit concept (BERGTHOLD, 1999).

Researches show that there's a relation between the types of representation and students' performances. Lauten et al. (1994) pointed out that there is a relationship between ideas of continuity and the functional representation that the participant is working with. The participant in their study handled equivalent problems diversely depending on whether the context was a graphical or analytic one. Bridgers (2007) found that most of the teachers who think continuity as important also find it difficult for students and see the continuity as a concept that students need to comprehend. These teachers notice that the link amongst a function being defined, a limit existing and the function being continuous are conflated in students' thinking. Students were found to prefer graphical representation when deciding the continuity of a function given both in algebraic and graphical forms.

The purpose of the study

Previous researches demonstrated that both teachers and students have misconceptions about limit and continuity concepts. The misconceptions and insufficient level of information of students about these concepts prevents the development of concepts of Calculus. Thus the development of limit and continuity concepts according to age and level of education should be investigated. Although there have been many studies on limit and continuity concepts and related difficulties (CORNU, 1991; FERRINI-MUNDY; GRAHAM, 1991; MAMONA-DOWNS, 2001; SZYDLIK, 2000; TALL; VINNER, 1981; DAVIS; VINNER, 1986; SIERPINSKA, 1987; AKBULUT; IŞIK, 2005; JORDAAN, 2005; WILLIAMS, 1991; HITT, 1994; BEZUIDENHOUT, 2001; HITT; PLANCHART, 1998; HITT; LARA, 1999; LAUTEN, ET AL., 1994; BRIDGERS, 2007; BURN, 2005; BERGE, 2006), few studies concentrated on the representations of these concepts and the relations between those representations (algebraic, graphical and verbal). From this point of view, this study aimed to reveal the way students and pre-service teachers define verbal, algebraic and graphical representations of function concept and to determine how function concept develops according to education.

Method

The Context of the Study

The schooling system in Turkey consists of three main components: basic education (elementary schools, age 6-14; 8 years), which is compulsory; secondary education (lycees or senior high schools, age 14-17; 4 years); and higher education (colleges and universities) (ÇALIK, 2005). The transitions between these levels are provided by central examinations. For example, a university entrance examination (called OSS) is required for student who wishes to continue to higher education after the completion of their secondary education. All schools throughout the country must use the same curricula, which was developed and implemented by the National Ministry of Education (TURKISH MINISTRY OF NATIONAL EDUCATION, 2005).

The first encounter of students with limit and continuity concepts is at the last year of high school and then pre-service teachers are taught these concepts in the General Mathematics Course in the first year of their university study and in the Calculus-I and Calculus-II Courses in their second years. Additionally, the students also face these concepts in the university entrance examination (OSS).

In the current paper, to abbreviate sample groups, some keynotes are exploited: Group A for grade 12 students, group B for first year pre-service mathematics teachers, group C for second year pre-service mathematics teachers and group D for third-grade mathematics student teachers.

Instruments and Data Collection Procedure

To determine students' conceptions and understandings in different grades, cross-age and longitudinal studies are generally used (COHEN; MANION; MARRISON, 2005). However, Abraham et al. (1992) stated that a cross-age study is more applicable than the longitudinal study when time is limited. Since the aim of this paper is to reveal concept development and the way limit and continuity concepts are understood by students from different levels of education, a cross-age study method was used. Because the students' levels of readiness to learn the concepts could be determined as a result of the cross-age studies (MORGIL; YÖRÜK, 2006).

In order to evaluate the students' understandings of limit and continuity concepts a test was developed including open-ended questions about verbal, algebraic and graphical representations of concepts. The developed test contains three main parts:

Some of the questions in the first part of the test (A1, A3, A5 and A7) and all questions in the second and third parts of the test are quoted from another study which aimed to reveal the difficulties of students about continuity concept (LAW, 1995). The other three questions pertaining to the verbal representation of limit concept (A2, A4 ve A6) were developed by the researcher in the light of expert views. In the first part of the test, there are three verbal expressions on limit concept and four on continuity concept and the students are asked to decide the correctness of these seven expressions and to explain their reasons. For example "*if left-hand limit and right-hand limit exist for a function at point x=a then the function has a limit at that point. Because ……*" The student is asked to decide whether this expression is correct and then explain the reasons. Similarly pertaining to the continuity concept, "*If a function can be written with a single formula, then it is continuous …. Because …..*" questions that will reveal the students' understandings were asked.

In the second set, the functions were by means of graphs. The students are asked to decide whether the four functions given graphically are continuous or have limits at the indicated points and explain their reasons. Identifying whether those functions were continuous and had limit at the point was not the main point of this study. I would rather focus on asking students to justify their answers. For example, the student is asked to decide whether the function whose graph was given below is continuous or have limit at the indicated point and give reasons.

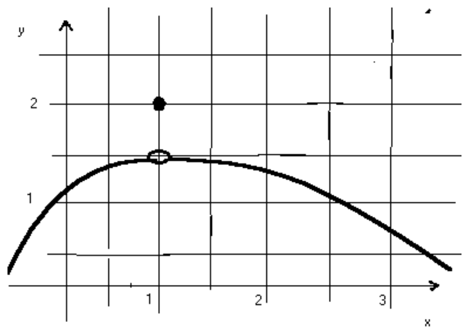

Figure 1

In the third part, the students are asked to determine whether the seven functions given algebraically are continuous or have limits at the indicated points and give reasons. For example, the student is asked to decide whether the

function given as g(x)= $\begin{cases} 2x+1, & x \geq 5 \\ 3x-4 & x < 5 \end{cases}$

is continuous and has limit at point x = 5.

Before administering the final form of the test, a group consisting of mathematics educators and mathematicians checked the validity of test items and agreed that the items were valid and appropriate to measure the students understanding of the limit and continuous concepts. They confirmed that scientific answers for questions presented were indeed appropriate.

The Sample

This study was performed with 268 pre-service teachers with the purpose of revealing these pre-service teachers' understandings of limit and continuity concepts and the development of those understandings. The sample comprising of totally 268 students with 61 grade 12 students, 73 first year pre-service mathematics teachers, 60 second year pre-service mathematics teachers and 74 third year pre-service mathematics teachers was randomly selected from two cohort secondary schools and department of elementary mathematics education at Karadeniz Technical University in the city of Trabzon in Turkey. The concepts were initially introduced to students at grade 12 and then the concepts are introduced at a more advanced level to the students at first and second years at the University.

Data Analysis

The following categorizations were used to analyze students' test responses to the qualitative (i.e., explanations) answers: a correct choice with the correct reason (3 point), correct choice with partial correct reason (2 point), incorrect choice (1 point) and no answer (0 points). The use of such classification criteria for student responses provides the researchers with an opportunity to compare students understanding. Differences among groups' scores in test were investigated by traditional statistical means using non parametric tests which are Kruskal-Wallis and Mann Whitney U because the data analyzed was categorical.

Results

The analyses and the interpretations of the scores obtained by the students from limit and continuity test will be presented in this section. Students' answers were categorized and were analyzed using SPSS software.

Table 1 shows the results related with the scores obtained by the students from the verbal, graphical and algebraic representations of limit concept.

Table 1: The means and standard deviations of the students related with limit concept.

Represenation	Groups	N	M	SD
Verbal	Group A	61	7,89	1,51
	Group B	73	7,33	1,50
	Group C	60	6,70	2,81
	Group D	74	5,19	2,40
Graphical	Group A	61	9,90	1,61
	Group B	73	10,23	1,76
	Group C	60	8,18	3,61
	Group D	74	5,96	3,17
Algebraic	Group A	61	13,42	3,63
	Group B	73	13,53	2,63
	Group C	60	11,73	5,77
	Group D	74	11,46	5,04

When the student scores obtained related with continuity concept are examined it can be seen that that while the performances of first year pre-service mathematics teachers were highest (respectively M = 8,06, SD = 2,69; M = 18,66, SD = 2,76), the performances of third year pre-service mathematics teachers were lowest (respectively M = 5,08, SD = 2,77; M = 11,90, SD = 5,93) in verbal and algebraic representations. In graphical representation, while second year pre-service mathematics teachers (M = 9,80, SD = 1,55) were found most successful, third year pre-service mathematics teachers (M = 7.34, SD = 2.80) were found least successful.

Table 4 shows the results of Kruskal-Wallis test performed according to the scores of students related with continuity concept.

Table 2: Results of Kruskal-Wallis test performed according to the scores of students related with limit concept.

Representation	Groups	N	Mean Rank	df	χ^2	p	Significant dif.
Verbal	Group A	61	169,53	3	41,63	.000	A-D, A-B, B-D, C-D
	Group B	73	145,53				
	Group C	60	140,45				
	Group D	74	89,92				
Graphical	Group A	61	161,92	3	72,07	,000	A-C, A-D, B-C, B-D C-D
	Group B	73	174,48				
	Group C	60	131,22				
	Group D	74	75,12				
Algebraic	Group A	61	145,57	3	8,20	,042	A-D, B-D C-D
	Group B	73	148,86				
	Group C	60	127,53				
	Group D	74	116,86				

A Kruskal-Wallis test was conducted to evaluate differences among students (grade 12 students, first year pre-service mathematics teachers, second year pre-service mathematics teachers and third-grade mathematics student teachers) about verbal, algebraic and graphical representation of continuity concept. The tests were respectively significant (χ^2 =41,63, p<0.05; χ^2 = 72,07, p<0.05; χ^2 =8,20, p<0.05). When the mean ranks of the groups are taken into account, 12th graders had the highest mean rank (169,53) for verbal representation of limit, whereas third year pre-service mathematics teachers had the lowest mean rank (169,53). In graphical representation, though, while the first year pre-service mathematics teachers (M = 174.48, SD = 148.86) were found most successful, the third year pre-service mathematics teachers (M = 75.12, SD = 116.86) were found least successful.

As a result of Mann Whitney U-test performed in order to determine the source of significant differentiation observed in the verbal, algebraic and graphical representations of limit concept, 12th graders, first year and second year pre-service mathematics teachers were found to be more successful than third year pre-service mathematics teachers (p<0.05). Likewise in the verbal representation 12th graders were found to be more successful than first year pre-service mathematics teachers (p<0.05). Furthermore, in the graphical representation 12th graders and first year pre-service mathematics teachers were found to be more successful than second year pre-service mathematics teachers ($p<0.05$).

Table 3 shows the results related with the scores obtained by the students from the verbal, graphical and algebraic representations of continuity concept.

Table 3: The means and standard deviations of the students related with continuity concept.

Representation	Groups	N	Mean	Std. Deviation
Verbal	Group A	61	7,43	3,06
	Group B	73	8,06	2,69
	Group C	60	6,90	2,90
	Group D	74	5,08	2,77
Graphical	Group A	61	8,80	2,47
	Group B	73	9,00	2,72
	Group C	60	9,80	1,55
	Group D	74	7,34	2,80
Algebraic	Group A	61	17,08	3,27
	Group B	73	18,66	2,76
	Group C	60	15,63	2,75
	Group D	74	11,90	5,93

When the student scores obtained related with continuity concept are examined it can be seen that while the performances of first year pre-service mathematics teachers were highest (respectively M = 8,06, SD = 2,69; M = 18,66, SD = 2,76), the performances of third year pre-service mathematics teachers were lowest (respectively M = 5,08, SD = 2,77; M = 11,90, SD = 5,93) in verbal and algebraic representations. In graphical representation, while second year pre-service mathematics teachers (M = 9,80, SD = 1,55) were found most successful, third year pre-service mathematics teachers (M = 7.34, SD = 2.80) were found least successful.

Table 4 shows the results of Kruskal-Wallis test performed according to the scores of students related with continuity concept.

Table 4: Results of Kruskal-Wallis test performed according to the scores of students related with continuity concept.

Representation	Groups	N	Mean Rank	df	χ^2	p	Significant dif.
Verbal	Group A	61	145,66	3	39,99	.000	A-D,B-C B-D, C-D
	Group B	73	168,05				
	Group C	60	136,91				
	Group D	74	90,25				
Graphical	Group A	61	136,33	3	28,57	.000	A-C, A-D C-D, B-D
	Group B	73	145,90				
	Group C	60	164,91				
	Group D	74	97,09				
Algebraic	Group A	61	150,74	3	73,34	.000	A-B, A-C A-D, B-C B-D, C-D
	Group B	73	187,38				
	Group C	60	117,35				
	Group D	74	82,86				

A Kruskal-Wallis test was conducted to evaluate differences among students (grade 12 students, first year pre-service mathematics teachers, second year pre-service mathematics teachers and third-grade mathematics student teachers) about verbal, algebraic and graphical representation of continuity concept. The tests were respectively significant (χ^2 = 39,99, p<0.05; χ^2 = 28,57, p<0.05; χ^2 = 73,34, p<0.05). When the mean ranks of the groups are considered, first year pre-service mathematics teachers (respectively 168.05; 187.38) have the highest mean rank, whereas the third year pre-service mathematics teachers have the lowest mean rank (90,25) in the verbal and algebraic representation of continuity. In graphical representation of continuity, while the second year pre-service mathematics teachers have the highest mean rank (164,91), the third year pre-service mathematics teachers have the lowest mean rank (97,09).

As a result of Mann Whitney U-test performed in order to determine the source of significant differentiation observed in the verbal, algebraic and graphical representations of continuity concept, 12^{th} graders, first year and second year pre-service mathematics teachers were found to be more successful than third year pre-service mathematics teachers (p<0.05). Likewise in the verbal and algebraic representation of continuity concept, first year pre-service mathematics teachers were found more successful than second year pre-service mathematics teacher ($p<0.05$). Moreover in the graphical and algebraic representation, 12th graders were found to be more successful than second year pre-service mathematics teachers. In the algebraic representation 12th graders were found to be more successful than first year pre-service mathematics teachers ($p<0.05$).

Discussion and Conclusion

When students' understandings of limit and continuity concepts are compared, the pre-service teachers in their 3rd year of study were found considerably less successful than other students in algebraic, verbal and graphical representation of limit and continuity concepts. The average scores of students and results of Mann Whitney U test performed show this result (Table 1 and 2). In the verbal representation of limit, 12th graders and in the algebraic and graphical representation first year pre-service mathematics teachers were found to be most successful. Furthermore, the understandings of 12th graders and first year pre-service mathematics teachers were found to be very close ($p<0.05$). In graphical representation of limit concept, 12th graders were found significantly more successful than second year pre-service mathematics teachers and second year pre-service mathematics teachers (M = 9,80, SD = 1,55) were found significantly more successful than third year pre-service mathematics teachers. This shows that the comprehension of graphical representation decreases with respect to education level of students. Previous studies on pre-service teachers (JORDAAN, 2005; AKBULUT; IŞIK, 2005, CORNU, 1991; SZYDLIK, 2000, ÇETIN, 2009) report meaning deficiencies about limit concept. This result aligns with the results of our study. The students stated that for a function given graphically to have limit, the function should be defined at that point. But if it was given as an algebraic partial function, they could calculate its limit easily. Therefore teaching limit concept using different representations of a function could prevent formation of misconceptions. Particularly for teaching the concept of limit, geometric illustrations are used as a possible approach.

When students' understandings of continuity concept are compared, the pre-service teachers in their 3rd year of study were found considerably less successful than other students in algebraic, verbal and graphical representation of limit and continuity concepts (Table 3 and 4). In the verbal and algebraic representations of continuity concept freshmen pre-service mathematics teachers were found to be more successful than all other students. Moreover it was found that this success is significant when compared to second year pre-service mathematics teachers. Moreover in the graphical and algebraic representation of continuity concept, 12th graders were found to be significantly more successful than second year pre-service mathematics teachers. Therefore it can be seen that the understandings of students of the algebraic and graphical representations of continuity concept decrease with the increase in years. Bezuidenhout (2001) and Vinner (1992) found that university students thought the function should be defined at the point where continuity is questioned. This result supports the result of our study that the conceptual comprehension of continuity concept decreases with respect to increase in level of education. Similarly in their study with secondary teachers Mastorides and Zacharlades (2004) found that some teachers could not read graphics of functions and transfer a verbal expression to symbolic expression correctly. This aligns with the result of our study that students' understanding of graphical representation of continuity concept decreases. The low level of comprehension of continuity concept by second year and third year pre-service teachers is the result of a number of major misconceptions. Specifically, the articulations of teachers derived from daily use such as the graph of a continuous function should be single part or its graph could be drawn without holding the pen up from the paper, may lead to misconceptions when the function is given in different representations. Since the idea that a function should be single part to be continuous is commonly adopted by students and pre-service teachers, when the function is given as algebraically partial, it was shown with this study that the success decreases with the increasing year of study. A similar result was also found in previous studies (HITT, 1994; HITT, 1998; HITT; PLANCHART, 1998; TALL; VINNER, 1981).

In the algebraic representation of continuity and limit concepts, first year pre-service mathematics teachers were found to be more successful than all other student groups. Furthermore, in the algebraic, graphical and verbal representations of continuity and limit concepts, third year pre-service mathematics teachers were found to be considerably less successful than all other student groups. On the other hand these students' success in the algebraic representation of limit concept is close to the successes of other groups.

Generally, freshman pre-service mathematics teachers were found to be more successful than all other student groups in different representations of limit and continuity concepts. This may be attributed to the Calculus Course taken by these students. Besides, the findings indicate that this performance decreases by time. In their study with secondary mathematics teachers, Mastorides and Zacharlades (2004) determined that the majority of teachers did not have deep understandings about limit and continuity concepts.

Pedagogical implications

The development of students' interpretation skills of different representations of functions should be emphasized to allow students to better understand the limit and continuity concepts. Therefore teaching should be enriched by prioritizing verbal, graphical and algebraic representations when designing instructional activities. Indeed it's easier than ever to access function graphics with the help of advanced technology and drawing software. Thus when examining limit of functions at indefinite or undefined points, it's possible to make use of different representations of functions. Moreover, the relationship between limit and continuity should also be stressed.

References

ABRAHAM, M.R.; GRYBOWSKI, E. B.; RENNER, J. W.; MAREK, E.A. Misunderstanding of Eighth Graders of Five Chemistry Concepts Found in Science. **Journal of Research in Science Teaching**, New York, v.29, p. 105 – 120, 1992.

AKBULUT, K.; IŞIK, A. Limit kavramının anlaşılmasında etkileşimli öğretim stratejisinin etkinliğinin incelenmesi ve bu süreçte karşılaşılan kavram yanılgıları. **Kastamonu Eğitim Dergisi,** 13(2), p. 497 – 512, 2005.

BERGE, A. Convergence of numerical sequences – a commentary on “the vice: some historically inspired and proof generated steps to limits of sequences” by R.P. Burn. **Educational Studies in Mathematics**, Dordrecht, Holanda, 61, p. 395 – 402, 2006.

BERGTHOLD, T. A. **Patterns of analytical thinking and knowledge use in students' early understanding of the limit concept**. Unpublished Doctoral Theses, University of Oklahoma, 1999.

BEZUIDENHOUT, J. Limits and continuity: some conceptions of first-year students. **International Journal of Mathematical Education in Science and Technology,** London, 32(4), p. 487 – 500, 2001.

BURN, B. The vice: Some historically inspired and proof-generated steps to limits of sequences. **Educational Studies in Mathematics**, Dordrecht, Holanda 60, p. 269-295, 2005.

BRIDGERS, L. C. **Conceptions of continuity**: An investigation of high school calculus teachers and their students. Dissertation theses. Syracuse University, 2007.

COHEN, L; MANION, L; MARRISON, K. **Research Methods in Education,** London: Taylor & Francis, 2005.

CORNU, B. Limits. In: TALL, D. (Ed.). **Advanced Mathematical Thinking.** Dordrecht / Boston / London: Kluwer Academic Publishers, 1991. p. 153 – 166.

ÇALIK, M. A cross-age study of different perspectives in solution chemistry from junior to senior High school. **International Journal of Science and Mathematics Education**, 3, p. 671 – 696, 2005.

ÇETIN, N. The performance of undergraduate students in the limit concept. **International Journal of Mathematical Education in Science and Technology,** London, 40(3), p. 323-330, 2009.

DAVIS R.; VINNER S. The Notion of Limit. Some Seemingly Unavoidable Misconception Stages. **Journal of Mathematical Behaviour**. v. 5, p. 281 – 303, 1986.

DUVAL, R. The cognitive analysis of problems of comprehension in the learning of mathematics. **Mediterranean Journal for Research in Mathematics Education,** 1(2), p. 1-16, 2002.

EVEN, R. Factors involved in linking representations of functions. **The Journal of Mathematical Behavior**, 17(1), p. 105-121, 1998.

FERRINI-MUNDY, J. ; GRAHAM, K. G. An overview of the calculus reform effort: issues for learning, teaching, and curriculum development. **American Mathematical Monthly**, Washington-US, 98 (7), p. 627-635, 1991.

HITT F.; PLANCHART O. Graphing of discrete functions versus continuous functions: A case study. Proceedings of North American **Chapter of the International Group of Psychology of Mathematics Education**, Raleigh, USA, p. 271 – 276, 1998.

HITT F. Teachers' Difficulties with the Construction of Continuous and Discontinuous Functions. **Focus on Learning Problems in Mathematics**, Framingham, Mass., US, v. 16, n. 4, p. 10 – 20, 1994.

HITT, F. ; LARA, H. Limits, continuity and discontinuity of fuction from two points of view: That of the teacher and that of the student. In Bills, L. (Eds.), **Proceedings of the British Society for Research into Learning Mathematics**, 19 (2), 1999.

HITT, F. Difficulties in the Articulation of Different Representations Linked to the Concept of Function. **Journal of Mathematical Behavior**, 17(1), p. 123 – 134, 1998.

JANVIER C. (Ed.). **Problems of Representations in the Teaching and Learningof Mathematics**. Hillsdale, NJ, USA: Lawrence Erlbaum, 1987.

JORDAAN, T. **Misconceptions of the limit concept in mathematics course for engineering students**. Unpublished Theses, University of South Africa, 2005.

KAPUT, J. Technology and mathematics education. In: GROUWS, D. A. (Ed.), **Handbook of research on mathematics teaching and learning,** New York: Macmillan, 1992. p. 515 – 556.

KAPUT, J. Democratizing access to calculus: New routes to old roots. In: SCHOENFELD, A. (Ed.), **Mathematical thinking and problem solving**. Hillsdale, NJ, USA: Lawrence Erlbaum, 1994. p. 77 – 156.

LAUTEN, A. D.; GRAHAM, K. G.; FERRINI-MUNDY, J. Student understanding of basic calculus concepts: Interaction with the graphics calculator. **Journal of Mathematical Behavior**, 13(2), p. 225 – 237, 1994.

LAW, C.H. Cognitive difficulties related to the understanding of continuity, **Proceding of the National Science Council**, Republic of China, Part D Mathematics, Science and Techonogy education, v. 5 (1), 1995.

MAMONA-DOWNS, J. Letting the intuitive bear on the formal; a didactical approach for the understanding of the limit of a sequence, **Educational Studies in Mathematics**, Dordrecht, Holanda,48(2/3), p. 259 – 288, 2001.

MASTORIDES, E.; ZACHARIADES, T. Secondary mathematics teachers' knowledge concerning the concept of limit and continuity, **Proceedings of the 28th conference of the international group for the Psychology of mathematics education**, vol 4, p 481 – 488, 2004.

MORGIL, I.; YÖRÜK, N. Cross-Age Study of The Understanding of Some Concepts In Chemistry Subjects In Science Curriculum, **Journal of Turkish Science Education**, Trabzon, Turkey, 3 (1), p. 15 – 27, 2004, 2006.

ÖZMANTAR, M. F. ; YEŞİLTEPE, S. Limit ve süreklilik konularında kavram yanılgıları ve çözüm arayışları, In: M. F. Özmantar et al. (Eds.), **Matematiksel Kavram Yanılgıları ve Çözüm Önerileri.** Ankara: Pegem Akademi, 2008, p. 181 – 221.

SCHOENFELD, A. (Ed.). **Mathematical Thinking and Problem Solving**. Hillsdale, NJ, USA: Lawrence Erlbaum, 1994.

SIERPINSKA, A. Humanities students and epistemological obstacles related to limits. **Educational Studies in Mathematics**, Dordrecht, Holanda,18, p. 371 – 397, 1987.

SZYDLIK, J. E. Mathematical beliefs and conceptual understanding of the limit of a function. **Journal for Research in Mathematics Education**, Reston, Va., US, 31(3), p. 258 - 276, 2000.

TALL D.; VINNER S. Concept image and concept definition in mathematics with particular reference to limits and continuity, **Educational Studies in Mathematics**, Dordrecht, Holanda,12, 151-169, 1981.

TURKISH MINISTRY OF NATIONAL EDUCATION. 2005. Web site: <http://ttkb.meb.gov.tr/ogretmen/>

VINNER, S. The function concept as a prototype for problems in mathematics learning. In: HAREL, G.; DUBINSKY, E. (Eds.), **The concept of function:** Aspects of epistemology and pedagogy. Washington, DC: Mathematical Association of America, 1992. (MAA Notes, Vol. 25, p. 195 – 213)

WILLIAMS, S. Models of limit held by college calculus students. **Journal for Research in Mathematics Education**, Reston, Va., US, 22 (3), p. 219 – 236, 1991.

ZIMMERMANN, W.; CUNNINHAM, S. (Eds.). **Visualization in Teaching and Mathematics**. Washington, DC, USA: Mathematical Association of America, 1991.

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DIAGNOSTIC TEST ABOUT LIMIT AND CONTINUITY

A: Answer the following questions as true or false and please explain your reason for choosing this answer.

A_1: If the limit of a function at a point $x = a$ *exists, then the function is continuous at that point..... Because…….*

A_2: If the left-hand limit and the right-hand limit of a function at a point $x = a$ exists, then the limit of the function exist at that point …….Because…….

A_3: If a function is defined at a point*, then the function is continuous at that point…….. Because……….*

A_4: limit of the function is a number or point which the function is get close but not reaches. ………Because…….

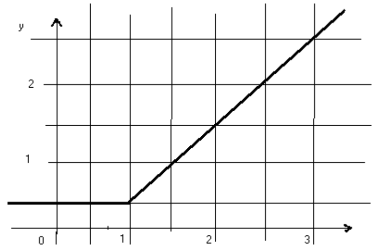
A_5: If the graph of a function is not broken at a point*, then the function is continuous at that point…….. Because……….*

A_6: The function must be defined at that point to exist the limit of a function a point……….. Because…………

A_7: If a function can be written as a single formula, is continuous….. Because………………...

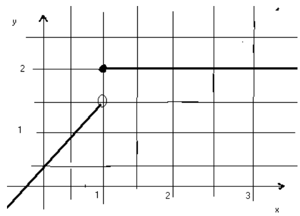
B: Answer the following questions as given graphics:

B_1:



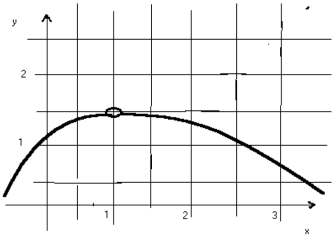
a) Is the function f in the above figure continuous at x=1? Explain.
b) Does $\lim_{x\to 1} f(x)$ exist? If so, please find the answer.

B_2:



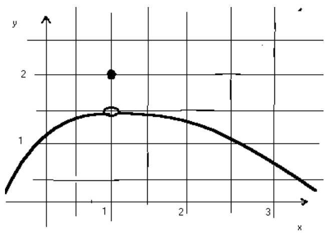
a) Is the function f in the above figure continuous at x=1? Explain.
b) Does $\lim_{x\to 1} f(x)$ exist? If so, please find the answer.

B_3:



a) Is the function f in the above figure continuous at x=1? Explain.
b) Does $\lim_{x\to 1} f(x)$ exist? If so, please find the answer.

B_4:



a) Is the function f in the above figure continuous at x=1? Explain.
b) Does $\lim_{x\to 1} f(x)$ exist? If so, please find the answer.

C: Answer the following questions:

C_1: $g(x)=x^3-8 \quad \text{if } x \neq 5$

a) Is the function g above continuous at x=5? Explain.
b) Does $\lim_{x\to 5} f(x)$ exist? If so, please find the answer.

C_2: $g(x)=\begin{cases} 2x+3, & x>5 \\ x+5, & x\le 5 \end{cases}$

a) Is the function g above continuous at x=5? Explain.
b) Does $\lim_{x\to 5} f(x)$ exist? If so, please find the answer.

C_3: $g(x)=\begin{cases} 4x-3, x\neq 5 \\ 15, x=5 \end{cases}$

a) Is the function g above continuous at x=5? Explain.
b) Does $\lim_{x\to 5} f(x)$ exist? If so, please find the answer.

C_4: $g(x)=\begin{cases} 2x+1, & x\ge 5 \\ 3x-4 & x<5 \end{cases}$

a) Is the function g above continuous at x=5? Explain.
b) Does $\lim_{x\to 5} f(x)$ exist? If so, please find the answer.

C_5: $g(x)=\dfrac{x^3-5x^2+x-5}{x-5}$

a) Is the function g above continuous at x=5? Explain.

b) Does $\lim_{x\to 5} f(x)$ exist? If so, please find the answer.